
Squiggle Ball Capture: A Simple, Visual Kinetic Theory Experiment

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When particles move about randomly in the presence of traps, how long does it take for them to be captured? Well, it depends on the average speed of the particles and the dimensions and distribution of the traps. For example, when neutrons are generated in nuclear fission reactions, they must be captured by other fissionable nuclei in order to sustain a chain reaction. But we learn in introductory physics that these energetic neutrons are traveling at enormous speeds and must be slowed to increase the time that they spend in the vicinity of the nuclei. While the capture of “thermal” particles into lower energy states is an important physical process, it is difficult to simulate macroscopically. For example, where do you get a collection of macroscopic objects that have the means to sustain random motion? Enter Squiggle Balls™—inexpensive spherical cat toys that use a battery-powered motor and asymmetric rotor to roll and tumble happily about.¹ Put a few on a bounded, elevated platform with several circular holes, and you have an ideal macroscopic system for exploring the world of capture.

One of the most fundamental features of the physical world is that objects prefer to occupy low energy states. This statistical tendency is observed in a wide range of scenarios from billiard games to electronic transitions in atoms. But how can we quantify the dynamics of the capture process? Students usually have a good intuitive sense of how capture works because of their experience with how ordinary things like billiard balls move under the force of gravity. However,

when the system is more abstract, their intuition is less effective. For instance, most introductory textbooks describe how moderators are used to slow the neutrons generated in fission reactions so that a chain reaction can proceed. How can we picture this behavior?

In recent years Squiggle Balls have been used to connect random macroscopic particle motion with a wide variety of thermal phenomena. Statistical analysis of Squiggle Ball motion can yield thermodynamic equations of state, equilibrium conditions, and occupational probability functions.^{2,3} In this paper, we use Squiggle Balls and gravity to explore the mechanism of capture. The simplicity of the phenomenon provides an excellent platform for honing the craft of experimental design. With very little physics background, students can take this project from start to finish: identifying key parameters, modeling the expected behavior, building and conducting the experiment, and interpreting results.

In our configuration, the Squiggle Balls move on one of two 1.5-m² bounded wooden platforms, the first containing four 25-cm diameter holes, and the second containing nine 13-cm diameter holes (see Fig. 1). Sometimes Squiggle Balls appear to favor rolling along the perimeter, so triangular wedges are mounted here to deflect the errant balls. The walls are positioned at half the distance between holes so that wall collisions mimic uninterrupted motion among uniformly spaced holes. Each of the holes can be covered to control the density of gravitational traps. For each density, four balls are released simultaneously from positions equidistant from the traps, the capture



Fig. 1. Photograph of the experimental setup. The Squiggle Balls roll about until they are “captured” by falling into one of the uncovered holes.

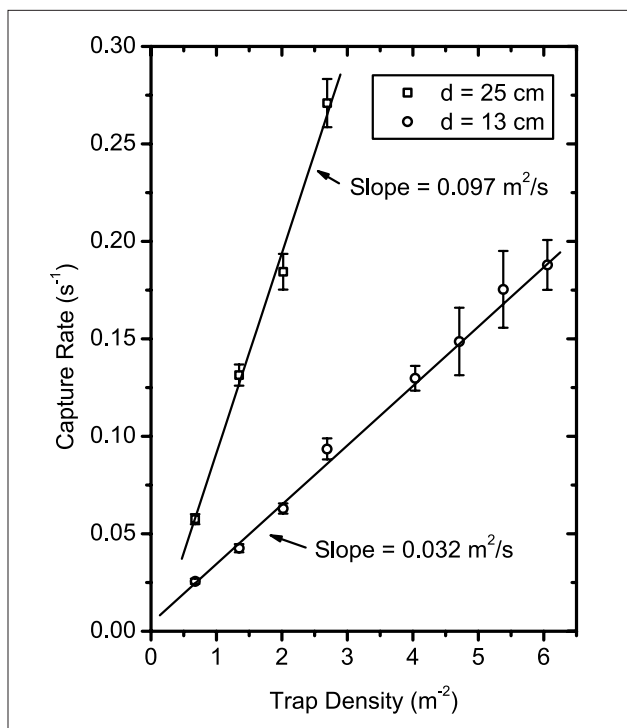


Fig. 2. Mean capture rate (the reciprocal of the average capture time) vs trap density for the hole diameters (d) given in the legend. Error bars are 90% confidence intervals computed using the standard error.

time for each ball is measured with a stopwatch, and the procedure is repeated approximately 25 times to collect ~ 100 measurements. We examine the average capture time τ versus trap density in the context of the following probabilistic model.

If we consider an ensemble of balls where the

probability of capture is constant in time, the rate of capture at any time t is proportional to number N of Squiggle Balls remaining:

$$\frac{dN(t)}{dt} = -rN(t),$$

where the rate $r = 1/\tau$ is the probability of capture per Squiggle Ball per second. Thus, N decays exponentially with time. For example, if $r \sim 0.10 \text{ s}^{-1}$, then in a sample of 12 balls, four or five should remain after 10 seconds. Squiggle Ball capture is comparable to many other physical mechanisms, like radioactive decay, that are governed by random processes.

The capture time of a thermal particle depends on three quantities: the thermal velocity of the particle, the distribution of the trapping centers, and the cross section of the traps. Since larger thermal velocities enable the particle to sample more space in less time, r should be proportional to the average particle velocity v . We also expect r to be proportional to the density n and cross section σ of the traps. The situation is a two-dimensional analog of the mean-free path computation in the kinetic theory of gases.⁴ In three dimensions, the path of a spherical object has a circular two-dimensional cross section, but motion in two dimensions only has a one-dimensional width. In addition, the ball only makes contact with the platform at a point, but the trapping dynamics are the same if we reverse the dimensions of the ball and the trap—so we *imagine* balls of diameter σ moving among point-sized traps. In the time interval Δt , these balls would travel a distance $v\Delta t$ and sweep out an area $v\Delta t\sigma$. If n is the trap density, then the number of trapping events equals the number of point-sized traps in the sample area: $v\Delta t\sigma n$. Hence, we arrive at $r = 1/\Delta t = nv\sigma$. Dimensional analysis confirms this relationship. In a two-dimensional system like ours, the density n has units of m^{-2} , and the cross section σ has units of m , yielding units of s^{-1} for r .

The average capture rate is plotted against trap density in Fig. 2. For each cross section, the slope of the linear fit should be equal to $v\sigma$. One might argue that v will depend on the number of open holes because fewer holes provide a longer “runway” to accelerate to higher speeds. However, when released, the Squiggle Balls accelerate to “steady-state” velocities in a small fraction of a second and move randomly thereafter.

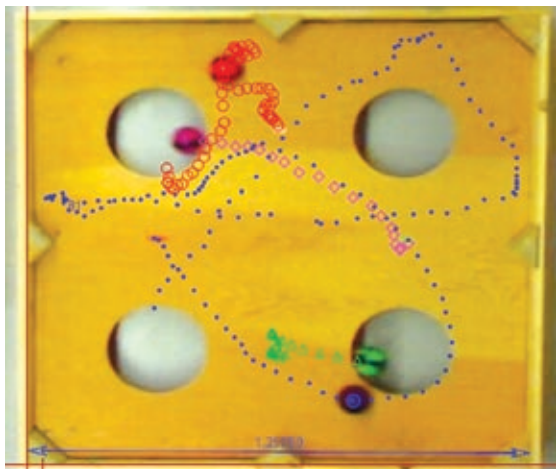


Fig. 3. Four representative Squiggle Ball trajectories on the four-hole platform with all traps open. The time interval between position measurements is 66 ms. Note the constant-speed centripetal motion about the lower right hole.

Since the fastest capture times are on the order of seconds, this ramp-up time is inconsequential. Further, with the exception of trapping events, the holes do not alter the magnitude of the velocity. For example, in near-capture events where the ball experiences centripetal acceleration as it rolls along an edge, the speed is not affected because the acceleration is perpendicular to the motion. We have used the video analysis program Tracker⁵ to follow the motion of four different Squiggle Balls (see Fig. 3) and to determine the average particle speed: $v = 0.50 \pm 0.04$ m/s. The standard deviation of the velocity distribution was nearly the same for all four balls, with an average of 0.25 m/s. Using $v = 0.50$ m/s, we obtain the effective cross sections $\sigma_{13} = 6.4$ cm for the 13-cm diameter hole and $\sigma_{25} = 19.4$ cm for the 25-cm hole.

These effective cross sections differ considerably from the hole diameters, with the smaller hole deviating by a comparable absolute amount and larger relative amount, suggesting that edge effects may be important. During off-center interactions, even when the center of the ball passes inside the edge of the hole, a capture event may not occur. We assume that $\sigma_{\text{effective}} = d - 2e$, where d is the actual hole diameter and e is the average distance from the edge of the hole that the center of the ball must reach to attain a 50% capture probability. This treatment yields $e = 3.1$ cm and 3.0 cm for the 13-cm and 25-cm diameter holes, respectively.

The edge effect analysis presented above yields consistent e values for the two values of d , but let's think about this more carefully. First, we note that when the diameter of the hole is smaller, the ball requires a larger centripetal acceleration to remain on the edge and retain the possibility for capture. Second, when the diameter of the hole is larger, the ball will have a longer transit time along the edge of the hole, enhancing the capture probability. (A good analogy can be made between this latter effect and the benefit of moderators in neutron capture. When the neutrons are moving more slowly, their interaction time with fissionable nuclei is increased.) In either case, the edge parameter e should decrease with increasing d .

So we made some additional measurements to estimate the value of e for each hole and discern which viewpoint is correct. Off-center interactions are simulated by launching inactivated balls down a linear track such that their velocities are approximately v and directing them at positions near the edge of the hole. We find $e \sim 2.4$ cm for the small hole and $e \sim 1.6$ cm for the large hole. The small hole clearly has a larger edge parameter, giving credence to the arguments made above. However, the ratio is smaller than the factor of 2 expected if e is inversely proportional to d , suggesting that the edge effect is more uniform than these simple arguments imply. In reality, the interaction of a ball with the edge of a hole is complex, as has been shown for putting in the game of golf.⁶ With Squiggle Balls, the situation is compounded by the fact that the center of mass is not coincident with the center of the ball and is free to move relative to the geometric center as the ball rolls.

As a final note, we'd like to point out that these experiments can be extended by tilting the platform to simulate a uniform applied field. If the lower elevation traps are covered so that only higher elevation traps are available, one would expect the capture time to increase with gravitational potential energy mgh in accordance with Boltzmann statistics. Furthermore, if obstacles rather than holes were placed on the tilted platform, Squiggle Balls moving from higher to lower elevation would simulate electrical conduction, with the obstacle dimensions and density controlling the resistivity of the system.⁷ The dependence of the average drift speed on tilt could be used to investigate the microscopic basis of Ohm's law.

Acknowledgments

The authors would like to thank Davidson students Kiril Simov and Blair Reynolds for performing the trap density-dependent capture time experiment, Professor Mario Belloni for reviewing the manuscript, and Instrumentation Specialist Jerry Snead for assistance with the edge effect and velocity measurements.

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PACS codes: 01.50.Pa, 05.00.00, 45.00.00

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